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Existence domains for nonlinear structures in complex two-ion-temperature plasmas

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Abstract
The existence domains for one-dimensional acoustic solitons and double layers in complex (dusty) plasmas with two ion temperatures are obtained, using the fluid dynamic paradigm with a general polytropic equation of state. Dust-acoustic solitons are considered in a four-component plasma of negative dust grains, cool and very hot ions, and very hot electrons. Whereas in a dust-ion-electron plasma only negative potential solitons are supported, the presence of a second ion component allows positive potential solitons to occur as well. The existence domain in parameter space is delineated, in particular, also for the reduced three-component case in which there are no free electrons, all electrons being adsorbed onto the dust grains. Next, the ion-acoustic regime is considered. Both positive and negative potential dust-ion acoustic solitons and double layers are found, and their existence conditions in the parameter space of cool ion density and Mach number derived.

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1. Introduction

Complex plasmas are characterized by the presence of massive dust grains (with, say, $m_d \simeq 10^{10} m_p$, where m_p is the proton mass) that carry relatively large charges (for example, $|Z_d| \simeq 10^4$), in addition to positive ions and electrons. Both positive and negative dust grains are found, but most studies have been based on the presence of negative dust grains, the more common scenario [1, 2].

Complex or 'dusty' plasmas are found in space and astrophysical environments, e.g. in the mesosphere, Saturn's rings, cometary regions and interstellar clouds [3]. A variety of laboratory experiments have been carried out on complex plasmas, e.g. studies of dusty plasma liquids and crystal structures, and waves and related phenomena [4–6]. There have also been significant theoretical developments, e.g. [7].

Three important effects arise from the presence of dust grains, namely the reduction of the number of free mobile electrons, as some of them are adsorbed on the grains, the introduction of a new timescale associated with motions (frequencies) based on the slower dust timescale, and the fact that the dust charge (and hence also the free electron density) may vary as a result of the oscillating electric fields during wave motion. The details of the charging process are not yet well understood, however, and are model dependent. For many purposes, the charging frequency is such that the charging process may be ignored. In what follows we shall use the approach of numerous studies of both linear and nonlinear waves in dusty (complex) plasmas, and assume that the dust grains are massive and have constant (negative) charge.

A complex (dusty) plasma can essentially support two types of acoustic waves, namely the higher frequency dust-modified ion-acoustic wave and the much lower frequency dust-acoustic mode. In the former, the inertia is provided by the ions, the (negative) dust grains are virtually immobile and their role is essentially one of reducing the number of free electrons. In the latter mode, the dust motion is vital, and the dust mass provides the inertia. These two forms of wave are similarly represented when one seeks acoustic solitons in such plasmas.

Recently, there has been a considerable development in our ability to delineate the existence domains for one-dimensional acoustic solitons in parameter space through the use of the fluid dynamic paradigm [8–10]. Not only does this approach emphasize the multi-fluid nature of the plasma, but through exploration of such important physical characteristics as the requirements of finite and non-negative densities of the different plasma components, and the role of the sonic points (defined below), one may delineate appropriate regions in which solitons (and double layers) may exist. This approach has recently been used to study solitons in a conventional dust-ion-electron plasma [11].

The sonic points, given by $v_{js} = V \big/ M_j^{2/(\gamma_j+1)}$, correspond to positions in space where the local flow speed of species j (i.e. v_j) matches its local acoustic speed, $(\gamma_j p_j/m_j n_j)^{1/2}$. Here γ_j is the polytropic index of species j, V is the equilibrium plasma flow speed in the wave frame, $M_j = V/c_{tj}$ is the species' Mach number and $c_{tj} = (\gamma_j p_{j0}/m_j n_{j0})^{1/2}$ is the species' thermal speed, with m_j, n_j, p_j the particle mass, number density and pressure of species j, respectively, while the subscript 0 denotes the relevant equilibrium value.

The sonic points limit the wave amplitude by choking the flow $(\mathrm{d}v_j/\mathrm{d}x \to +\infty)$. Analysis of the structure function obtained from the fluid equations—essentially the negative of the well-known Sagdeev pseudopotential—enables one to find the existence regions for solitons and double layers.

This paper deals with the effect of a two-temperature distribution of ions on the existence domains of these two types of acoustic solitons (and, where appropriate, double layers). We note, however, that we restrict ourselves to the situation in which the two ion components have the same mass and charge. The superposition of two Maxwellian distributions with different temperatures is often found to show a good fit to observed non-thermal velocity distributions, with an enhanced tail. This approach may be used, for instance, to describe solar wind protons or the conditions in a beam-plasma experiment.

In section 2 we outline the basic theory, which is then applied to the two frequency ranges (dust-acoustic and ion-acoustic) in sections 3 and 4, respectively. In the former case we consider a four-component plasma of dynamic negative dust grains, cool and very hot cool ions, and very hot electrons. The reduced case in which there are no free electrons is also discussed. For the ion-acoustic solitons, the model is based on a four-component plasma of hot and cold ions, very hot electrons and effectively immobile negative dust grains. In section 5 we present some conclusions.

Preliminary work on this topic was presented at the *International Conference on the Physics of Dusty Plasmas, 2005* [12].

2. Theory and equations

We begin with the two fluid equations for each species, namely the equations of continuity and momentum transfer, incorporating a polytropic equation of state for the pressure, coupled by Poisson's equation. In the frame moving with the nonlinear structure, the time derivatives vanish, and one obtains a set of Bernoulli-like conserved quantities from the fluid equations [10]. In particular, the equation of continuity implies that $v_j n_j = v_{j0} n_{j0} = n_{j0} V$, defined in terms of equilibrium (undisturbed) quantities, where $v_{j0} = V$ in the wave frame.

We follow the standard approaches [10, 13, 14] to obtain general expressions for the species' energy function, $\mathcal{E}_j$, and the pressure function, $\mathcal{P}_j$, respectively, for a fluid with polytropic index $\gamma_j \neq 1$,

$$\mathcal{E}_j \equiv \frac{1}{2}\left(v_j^2 - V^2\right) + \frac{c_{tj}^2}{\gamma_j - 1}\left(\frac{V^{\gamma_j - 1}}{v_j^{\gamma_j - 1}} - 1\right) = -\frac{q_j}{m_j}\phi \tag{1}$$

$$\mathcal{P}_j \equiv V(v_j - V) + \frac{c_{tj}^2}{\gamma_j}\left(\frac{V^{\gamma_j}}{v_j^{\gamma_j}} - 1\right). \tag{2}$$

An important feature of these equations is the equality of $m_j \mathcal{E}_j / q_j$ that is observed from the set of equations (1). This enables one to substitute in $\mathcal{E}_j$ for one species in terms of the velocity of another species or in terms of the local electrostatic potential ϕ.

The global momentum invariant or wave structure function, $\mathcal{R}$, is

$$\mathcal{R} \equiv \sum_j n_{j0} m_j \mathcal{P}_j = \sum_j n_{j0} m_j (\mathcal{P}_j - \mathcal{E}_j) = \frac{\varepsilon_0}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}x}\right)^2, \tag{3}$$

where the symbols have their usual meanings, and we have used the fact that $\sum_j n_{j0} m_j \mathcal{E}_j = -\phi \sum_j n_{j0} q_j = 0$. Using (1), we shall write the structure function either in terms of ϕ or in terms of the flow velocity of a specific species, v_ℓ.

This function, which, within a numerical factor, is the negative of the 'Sagdeev' pseudopotential commonly used in soliton studies [15], governs the soliton behaviour. In particular, the existence of a soliton or a double layer depends on two important conditions on the behaviour of $\mathcal{R}$.

On the one hand, there is a minimum requirement, namely that at the 'origin' (suitably defined), the function satisfies the necessary soliton condition. When $\mathcal{R}$ is written as a function of ϕ, this is $\mathcal{R}(0) = \mathcal{R}'(0) = 0, \mathcal{R}''(0) > 0$. For $\mathcal{R} \equiv \mathcal{R}(v_\ell)$, the 'origin' is at $v_\ell = V$. The soliton condition is equivalent to the requirement that the flow be supersonic (i.e. relative to the speed of the acoustic wave under consideration). From Poisson's equation we note that $\mathcal{R}'$ is made up of contributions of the charge densities of the different species.

The second condition is the presence of a root of the equation $\mathcal{R} = 0$ within the permissible range of the velocity of a specific species v_ℓ or the potential ϕ. In practice, this range is restricted by the sonic points or other limiting features, such as the vanishing of the density or velocity of a species. The form of the limiting feature depends on the problem under scrutiny and examples will be discussed in detail for the two specific cases dealt with below.

The conditions underlying the limits will be found to yield existence criteria for solitons in an appropriate parameter space.

3. Dust-acoustic solitons

In the low-frequency dust regime, we shall consider a complex plasma made up of four components, namely massive negative dust grains of constant charge, flowing supersonically (i.e. satisfying $c_{td} < V$), and cool (subscript c) and hot (h) ions of the same mass and charge, together with hot electrons (subscript e), all flowing subsonically, i.e. with $V < c_{tc}, c_{th}, c_{te}$. As a special case, we shall also deal with the situation in which all electrons are adsorbed onto the dust grains, and there are no free electrons.

To study solitons in the dust regime, we use the ordering $c_{td} \ll V \ll c_{tc} \ll c_{th}, c_{te}$. Our discussion is thus restricted to the case in which $T_c \ll T_h$, where $T_{c,h}$ represent the effective cool and hot ion temperatures, respectively, and T_e, the electron temperature, is also sufficiently high to ensure that $c_{tc} \ll c_{te}$. The assumption that the hot ions and the electrons are 'super-hot' enables us to linearize their contributions to the structure function, $\mathcal{R}$. We note that the two ion sonic points satisfy $v_{hs,cs} \to \infty$, with $v_{hs} > v_{cs}$.

The heavy dust particles contribute only an inertial term, while the electrons and the ions contribute only thermal terms to $\mathcal{P}$ and $\mathcal{E}$, and hence to $\mathcal{R}$.

For this problem it is useful to work with the electrostatic potential as the independent variable. We find it convenient to normalize the structure function $\mathbf{R} = \mathcal{R}Z_d/n_{i0}m_dV^2$, where the total ion density is $n_{i0} = n_{c0} + n_{h0}$, and introduce the normalized potential $\psi = Z_d e\phi/m_d V^2$, where the dust grain charge is $q_d = -Z_d e$. Defining the cool ion fraction $\alpha = n_{c0}/n_{i0}$, and the dust fraction of the negative charge $\delta = Z_d n_{d0}/n_{i0}$, yields a fractional hot ion density $(1-\alpha)$ and a fractional electron density $(1-\delta)$. We also define the dust-cool ion Mach number, $M^2 \equiv M_{dc}^2 = m_d V^2/m_i Z_d c_{tc}^2$, and we shall write $\gamma = \gamma_c$, as both γ_h and γ_e play no role when the hot ions and electrons are 'super-hot'.

The normalized structure function is then

$$\mathbf{R}(\psi) = \delta[\sqrt{1+2\psi} - 1] - [(1-\alpha)\psi] + [(1-\delta)\psi] + \left[\frac{\alpha}{\gamma M^2}\{[1-(\gamma-1)M^2\psi]^{\gamma/(\gamma-1)} - 1\}\right]. \tag{4}$$

Here, the first term is due to the dust, and the second and third linearized terms are due to the 'super-hot' hot ions and electrons, respectively, while the last term arises from the cool ions. It will be noted that the two linear terms combine to yield an effective contribution from the very hot components of the form $(\alpha-\delta)\psi$. Thus this contribution depends only on the difference between the cool ion and dust charge densities.

As the resulting terms in $\mathbf{R}(\psi)$ are proportional to either α or δ, one can simplify the discussion by dividing through by δ and introducing a new parameter $f = \alpha/\delta = n_{c0}/Z_d n_{d0}$, the ratio of the cool ion charge density to the dust charge density. The renormalized structure function may then be written as

$$R(\psi) = [\sqrt{1+2\psi} - 1] - [(1-f)\psi] + \left[\frac{f}{\gamma M^2}\{[1-(\gamma-1)M^2\psi]^{\gamma/(\gamma-1)} - 1\}\right]. \tag{5}$$

Apart from a change of sign of the potential, this is mathematically the same form as one finds for the structure function for ion-acoustic solitons in a two-electron-temperature plasma [13], and thus the dust-acoustic solitons found here are analogues of the ion-acoustic solitons obtained in that context. However, care must be taken in their interpretation.

The structure function also has similarities to that found for dust-acoustic solitons in a dust-ion-electron plasma [11].

The dust-acoustic speed in the four-component complex plasma is given by

$$V_{da}^2 = \{\delta Z_d/m_d\}\{[\alpha/T_c] + [(1-\alpha)/T_h] + [(1-\delta)/T_e]\}^{-1}. \tag{6}$$

It follows that for $T_h, T_e \gg T_c$,

$$V_{da}^2 \simeq Z_d T_c / f m_d. \tag{7}$$

Hence, one may show that the soliton condition yields $M^2 > M_s^2 = 1/f$ in the approximation of hot ions and electrons that are 'super-hot'. For the case of no free electrons ($\delta = 1$), this simplifies to the usual expression found in the analogous ion-acoustic problem, namely $M_s^2 = 1/\alpha$ [13].

To explore the upper limit on the Mach number, we first consider negative potential solitons. These are limited by the first term—infinite compression of the dust grains at $\psi = -1/2$ coincides with the dust sonic point, $u_{ds} = 0$. Substituting this value of ψ in (5) leads to a condition, $M < M_n$, where the upper limit for negative potential solitons, M_n, satisfies

$$\left[1 + (1/2)M_n^2(\gamma - 1)\right]^{(\gamma/(\gamma-1))} = 1 + \gamma M_n^2(1 + f)/2f. \tag{8}$$

As an illustration, for $\gamma = 2$, this expression yields $M^2 < 4/f$. For the reduced three-component dusty plasma ($\delta = 1$), the expression satisfied by M_n reduces to that found for the analogous ion-acoustic situation [13].

Next we turn to the possibility of positive potential solitons, which are indeed found to occur, in contrast to the situation for a dust-ion-electron plasma [11]. Their range of occurrence is limited by the cool ion 'lid', at which $n_c \to 0$. From the cool ion charge density term in $R'(\psi)$ this is found to occur at the 'lid potential', $\psi_\ell = 1/(\gamma - 1)M^2$. Clearly, this only applies for $\gamma \neq 1$.

Substitution in (4) leads to

$$\sqrt{1 + \frac{2}{(\gamma - 1)M^2}} < 1 + \frac{1 - f/\gamma}{M^2(\gamma - 1)}, \tag{9}$$

from which one obtains an upper limit on M for positive potential solitons, given by $M^2 < (\gamma - f)^2/2f\gamma(\gamma - 1) \equiv M_p^2$. Again, for the case in which $\delta = 1$, this reduces to the expression for the limit M_p in the analogous ion-acoustic case [13]. Combining this result with the soliton condition $M > M_s = 1/f$ yields a critical ratio of cool ion density to dust charge density, $f_c = \gamma - \sqrt{2\gamma(\gamma - 1)}$ (see [13] for the case $\delta = 1$), below which positive potential solitons will not occur. Hence it follows that in this model, positive potential solitons can only occur in a plasma with $\gamma < 2$.

As we have noted, the above calculation has assumed that $\gamma \neq 1$. For negative potential solitons the expressions go over seamlessly to the isothermal case, $\gamma = 1$, when one takes the appropriate limits. However, for positive potential solitons there is no 'lid', i.e. n_c does not vanish for any finite value of ψ, and there is thus no upper limit on M, other than due to a breakdown of the underlying model.

In figure 1, we plot the existence domain for dust-acoustic solitons in the parameter space, (f, M), for a typical value of the polytropic index, $\gamma = 1.5$. Although this value has no particular physical significance, it is nevertheless useful as it is in the range $1 < \gamma < 2$, and leads to mathematical simplifications. In interpreting the figure, one must bear in mind that there are two possible interpretations. In general, the variable f represents the ratio of the cool ion charge density to the dust charge density, $f = \alpha/\delta = n_{c0}/Z_d n_{d0}$, in a four-component plasma which includes super-hot electrons, as indicated above. It may, however, also be interpreted as the cool ion density fraction in a three-component complex plasma involving no free electrons. This is equivalent to the use of the notation f in previous related papers, e.g. [13].

It is seen that in the light grey region only negative potential solitons are found. Positive potential solitons are found in both the dark grey region and, together with negative potential

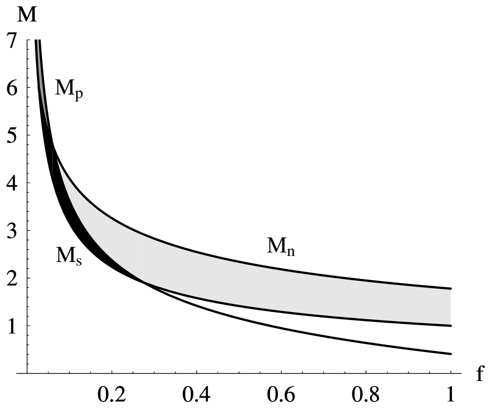

Figure 1. Existence domain in the parameter space (f, M): regions in which the dust-acoustic solitons that are found in a plasma with a 'typical' polytropic index value $\gamma_c = 1.5$ have exclusively a negative potential (light grey) or positive potential (dark grey). Both types of soliton occur in a narrow overlap region (black). Here, M is the dust-cool ion Mach number and f is the ratio of the cool ion density to the dust charge density. In the special case in which there are no free electrons, f represents the cool ion density fraction.

solitons, in the black region. This result differs from that of a dusty plasma with a single ion temperature [11], in which only negative potential solitons are found.

4. Ion-acoustic-like solitons

To investigate nonlinear structures in the ion regime, we assume an ordering of the form $c_{td} \ll c_{tc} \ll V < c_{th} \ll c_{te}$. Thus, in particular, the cool ions are deemed to be flowing supersonically (i.e. $V > c_{tc}$) and the hot ions subsonically (i.e. $V < c_{th}$). Based on the ordering, we restrict ourselves to a situation in which the cool ions are not just supersonic, but are cold, arising from which it follows that their sonic point is at $v_{cs} \to 0$.

For this problem, it is convenient to write the structure function, $\mathcal{R}$, in terms of the hot ion speed v_h. We shall also find it useful to write $\mathcal{R}$ in the alternative form given in (3), i.e. in terms of a sum over $(\mathcal{P}_j - \mathcal{E}_j)$.

We note that, on the ion timescale, the dust grains may be regarded as super-massive, enabling us to linearize their contribution to $\mathcal{R}$. The resultant approximation that $v_d \simeq V$ leads to $\mathcal{E}_d \simeq \mathcal{P}_d$ to order m_p/m_d. For the super-hot electrons, $v_e \simeq V$, too, and hence $\mathcal{E}_e \simeq \mathcal{P}_e$ to order c_{th}/c_{te}. Thus neither of these two species makes a contribution to the structure function as written in (3).

An interesting point that follows from this result is that, in this model, and hence within the restrictions listed, there is no constraint on the electron and dust grain densities—the results that follow apply for an arbitrary sharing of charge between these two negative species.

In addition, we note that as the cool ions are cold, only the inertial terms survive in the energy and momentum expressions for the cool ions, $\mathcal{E}_c$ and $\mathcal{P}_c$, respectively.

However, it is important to note that we retain the full expressions for the hot ions, that is, both inertial and thermal terms in $\mathcal{E}_h$ and $\mathcal{P}_h$ are included. This differs from models that are commonly used, in which the inertial contributions for the light particles are ignored, as, for instance, in section 3, in [8–11] and in [15]. A very common assumption is that the light particles may be represented by a Boltzmann distribution. This not only implies that

$\gamma_{h,c} = 1$, but also that the ions are inertialess. Although clearly $m_i/m_d \ll 1$, it will turn out that the finite ion mass may in fact play an important role, as observed previously with respect to dust-acoustic double layers [16]. Similarly, the existence domain of ion-acoustic solitons in a two-electron-temperature plasma was found to be affected by finite cool electron inertia [17].

We now write the cool ion fraction $f = n_{c0}/n_{i0}$, and define the hot ion Mach number $M = V/c_{th} < 1$. These two parameters will make up the parameter space in which we examine existence domains.

Writing $u_j = v_j/V$ and $R = \mathcal{R}/n_{i0}m_i V^2$ (with m_i the ion mass), as in [14], we normalize the expressions for $\mathcal{E}_j$, $\mathcal{P}_j$, and $\mathcal{R}_j$, use (1) to eliminate v_c, and write $\gamma \equiv \gamma_h$, as γ_c and γ_e play no role to obtain

$$R(u_h) = f P_c + (1-f)P_h - E_h, \tag{10}$$

where P_c, P_h and E_h are the appropriate normalized functions. Explicitly, we may write

$$\begin{aligned} R(u_h) = f &\left[\sqrt{u_h^2 + \frac{2}{(\gamma-1)M^2}\left(\frac{1}{u_h^{\gamma-1}} - 1\right)} - 1\right] \\ &+ (1-f)\left[u_h - 1 + \frac{1}{\gamma M^2}\left(\frac{1}{u_h^{\gamma}} - 1\right)\right] \\ &- \left[\frac{1}{2}\left(u_h^2 - 1\right) + \frac{1}{(\gamma-1)M^2}\left(\frac{1}{u_h^{\gamma-1}} - 1\right)\right]. \end{aligned} \tag{11}$$

This expression is very similar to that found earlier [14] in the study of electron-acoustic solitons in a two-electron-temperature plasma. As may be deduced from (1), the essential difference lies in the sign of the potential, which arises because of the change of sign of the charges from the two-electron case to the present two-ion case.

Hence, it follows that the model under consideration yields ion-acoustic-like solitons and double layers in the dust environment, that are direct analogues of electron-acoustic structures in a conventional two-electron-temperature plasma, subject to an appropriate change of sign of potential of the electrostatic structure.

As our analysis follows that given in detail earlier [14], we shall present it in outline only. For a solitary wave to be formed, the necessary soliton condition, which in this case becomes $R(1) = R'(1) = 0$, $R''(1) > 0$, must be satisfied. This condition immediately leads to the lower limit on the Mach number, $M^2 > f$, as found in the analogous case [14]. We recall that here M is not the usual Mach number associated with the sound speed in the medium, but is the 'hot ion Mach number' $M = V/c_{th}$.

Within the model, it follows that the maximum hot ion Mach number must satisfy $M < 1$, because of the original ordering. However, for a given value of f, a lower cut-off may exist. The cut-offs for M may be found from one of the following limiting features which reduce the admissible domain of R,

(i) The cool ion sonic point is reached, i.e. $u_c = u_{cs} = 0$, for a value of $u_h \neq u_{hs}$, the hot ion sonic point. From the continuity constraint, this coincides with the infinite compression of the cool ions ($n_c \to \infty$).

(ii) It can be shown [14], that for $M > M_A$, where $M_A^2 = (2/[\gamma+1])^{(\gamma+1)/(\gamma-1)}$, the hot ion sonic point u_{hs} is reached for a finite cool ion flow speed, $u_c > u_{cs} = 0$.

(iii) A third possibility is that, before a sonic point is reached (i.e. for $u_c \neq u_{cs}$ and $u_h \neq u_{hs}$), a double layer is formed as the limiting case of a set of solitons [15]. A double

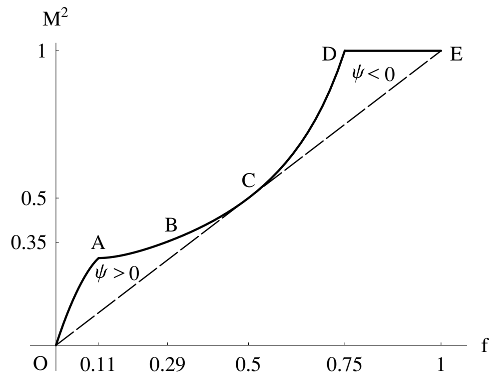

Figure 2. Existence domains: positive and negative potential ion-acoustic-like solitons and double layers for $\gamma_h = 2$. Soliton limits in different sections of the bounding curve arise from different physical effects (see text). Here, M is the hot ion Mach number and f the cool ion density fraction.

layer, representing adjacent oppositely-charged layers, satisfies $R(u_{hm}) = R'(u_{hm}) = 0$, $R''(u_{hm}) > 0$, where u_{hm} is the maximum soliton amplitude for the specific value of f.

In figure 2, we present a typical existence diagram for ion-acoustic solitons in a complex plasma in the parameter space (f, M^2) for the analytically simple polytropic index $\gamma = 2$. Although this value does not have any particular physical significance, it leads to analytically simple structures for R, and is typical of a set of values of γ. In the analogous study of electron-acoustic solitons, it was shown [14] that essentially similar figures are found for other values, such as $\gamma = 1$ and $\gamma = 3$.

We note that solitons occur only for $M^2 > f$, but that in this model they are not limited to low values of M^2 or f. In fact two distinct sub-regions are found, separated at point C, i.e. at $M^2 = f_c = 3/(\gamma + 4)$ [14], which for $\gamma = 2$ places the critical point at $M^2 = f_c = 0.5$. As indicated in the figure, positive potential solitons, i.e. with $\psi > 0$, occur in region OABC, and negative potential solitons in region CDE.

As may be deduced from the analogous electron-acoustic case [14], the cool ion sonic point limits the soliton amplitudes along the bounding curve OA, while the hot ion sonic point is the limiting feature along AB. Finally, double layer formation limits soliton amplitudes along BC. Over the range OABC, for $f < f_c$, the solitons have positive potentials. The soliton potentials are reversed for $f > f_c$. Along CD negative potential solitons are bounded by double layer formation, and along DE by the model assumption $M^2 < 1$ [14].

5. Discussion and summary

First, we return to the question of our neglect of dust grain charging. As pointed out earlier, these effects are often neglected, because the models are still the subject of some controversy (see, for instance, [18]), the results tend to be model dependent, and depending on detailed plasma conditions, the charging rate is such that only negligible effects arise. Thus, for relatively low dust grain density the charging effect is found to be unimportant for dust-acoustic solitons, while for high grain density the existence region may be reduced in size, details depending on the ion distribution function [19]. Alternatively, as the charging process is a dissipative effect, it can lead to soliton damping in some plasmas, preventing observation on a longer timescale (e.g. for dust-acoustic solitons [20]).

A brief comment on the use of the existence diagrams is appropriate. They could be useful both in regard to the interpretation of electrostatic spikes or other persistent wave-like structures obtained from satellite-based experiments, and to the setting up of laboratory experiments. In the former case, one would calculate from measured plasma data the coordinates in the given parameter space, and depending on whether they lie within the existence domain or not, deduce whether the observed phenomena could be explained through these dust-acoustic or ion-acoustic-like solitons. On the other hand, if one wished to generate in the laboratory either of the two soliton types discussed here, the existence diagrams could guide one in the choice of experimental parameters.

In summary, we have investigated acoustic solitons in a two-ion-temperature complex plasma, and considered dust-acoustic and ion-acoustic-like solitons. These are analogues of equivalent regimes for ion-acoustic and electron-acoustic solitons in two-electron-temperature plasmas, respectively. Subject to allowing for a change of sign of the soliton potential and the associated rarefaction (compression), the existence diagrams obtained previously [13, 14] carry over to the dusty plasma models considered. Hence, one finds existence domains both for positive and negative potential ion-acoustic-like solitons and double layers, and also for dust-acoustic solitons of both potential signs. The latter differs from the case of a dust-ion-electron plasma, where only negative potential solitons are supported [11].

Finally, while we have assumed in the calculation that the two ion species have the same mass, it is likely that the existence domains would also be approximately valid for the case of the two species having a mass ratio in the vicinity of 1. Thus, the results could also be used to model approximately the soliton domains for a dusty plasma involving, say, oxygen and nitrogen or oxygen and water vapour positive ions.

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